DESIGN OF DC MACHINE
OUTPUT EQUATION

\[ P_a = \text{power developed by armature in kW} \]
\[ P = \text{rating of machine in kW} \]
\[ E = \text{generated emf, volts; } V = \text{terminal voltage, volts} \]
\[ p = \text{number of poles; } I_a = \text{armature current, A} \]
\[ I_z = \text{current in each conductor, A} \]
\[ a = \text{number of parallel path; } Z = \text{number of armature conductor} \]
\[ N = \text{speed in rpm; } n = \text{speed in rps} \]
\[ D = \text{armature diameter, m; } L = \text{core length, m} \]
\[ \Phi = \text{flux per pole, weber; } \tau_p = \text{pole pitch} \]

\[ P_a = \text{power developed by armature in kW} \]
\[ = E \times I_a \times 10^{-3} \]

And \( E = p \Phi Z n/a \)
Thus \[ P_a = \left( \frac{p \Phi Z n}{a} \right) a x I_a x 10^{-3} = (p\Phi) \left( \frac{I_a Z}{a} \right) n x 10^{-3} \]
\[ = (p\Phi) (I_z Z) n x 10^{-3} \quad \text{since } I_a/a = I_z \]

Now \( p\Phi \) = total magnetic loading

And \( B_{av} = \frac{(p\Phi)}{(\pi D L)} \) or \( p\Phi = B_{av} \pi D L \)

\[
\text{ac} = \text{specific electric loading} = \left( \frac{I_z Z}{\pi D} \right)
\]

or \( I_z Z = \text{ac} \pi D \)

From above equations

\[
P_a = (B_{av} \pi DL) (ac \pi D) n x 10^{-3}
\]
\[ = (\pi^2 B_{av} \text{ ac} 10^{-3}) D^2 L n
\]
\[ = c_o D^2 L n \quad \text{where } c_o = \pi^2 B_{av} \text{ ac} 10^{-3} = \text{output coefficient}
\]

Also

\[
D^2 L = \left( \frac{1}{c_o} \right) \left( \frac{P}{\eta} \right) \quad Pa = \frac{P}{\eta} \quad \eta = \text{efficiency of machine}
\]
**Estimation of Pa:**

In case of generator

\[ \text{\( P_a \)} = \text{input power} - \text{rotational losses} \]

\[ = \left( \frac{\text{output power}}{\text{efficiency}} \right) - \text{rotational losses} \]

\[ = \frac{P}{\eta} - \text{rotational losses} \]

Rotational losses = friction, windage and iron losses

In case of motor

\[ \text{\( P_a \)} = \text{output power} + \text{rotational losses} \]

\[ = P - \text{rotational losses} \]

In case of large machines very small difference between \( P \) and \( P_a \). So friction, windage and iron losses could be neglected.

\[ \text{\( P_a \)} = \frac{P}{\eta} \text{ for generator} \]

\[ \text{\( P_a \)} = P \text{ for motor} \]
In case of small machines friction, windage and iron losses cannot be neglected.

Assume friction, windage and iron losses = 1/3 (total losses)

Total losses = input power – output power

\[ = \frac{P}{\eta} - P = \frac{P(1- \eta)}{\eta} \]

Hence friction, windage and iron losses = \( \frac{P(1- \eta)}{3\eta} \)

For small motors

\[ P_a = P + (\text{friction, windage and iron losses}) \]

\[ P_a = P + \frac{P(1- \eta)}{3\eta} = \frac{P(1+2\eta)}{3\eta} \]

For small generators

\[ P_a = \frac{P}{\eta} - (\text{friction, windage and iron losses}) \]

\[ = \frac{P(2+\eta)}{3\eta} \]
Choice of specific magnetic loading (Bav):

Flux density in teeth: if a high value of flux density is assumed for air gap, the flux density in armature teeth also becomes high. The maximum value of flux density in the teeth at minimum section should not exceed a value of 2.2 \( \text{wb/m}^2 \) because at higher flux density i) increased iron losses and ii) higher ampere turns requires for passing the flux through teeth leading to increase copper losses and cost of copper.

Frequency: the frequency of flux reversal in the armature is given by \( f = np/2 \). Higher frequency will result increased iron losses in the armature core and teeth. So there is a limitation in choosing higher \( B_{av} \) for a machine having higher frequency.

Voltage: for high voltage machine space required for insulation is large. Thus for a given diameter less space is available for iron leading to narrower teeth. Therefore lower value of \( B_{av} \) has to be taken otherwise teeth flux density increases beyond the permissible limit.

Value of \( B_{av} \) varies from 0.4 to 0.8 \( \text{wb/m}^2 \).
Choice of specific electric loading (ac):

*Temperature rise:* A higher value of ‘ac’ results in a high temperature rise of windings. A high value of ‘ac’ can be used for machine using insulating material which withstand high temperature rise.

*Speed of machine:* for high speed machine, the ventilation is better and greater losses could be dissipated. Thus a higher value of ‘ac’ can be used for higher speed machine.

*Voltage:* machine with high voltage require large space for insulation, therefore there is less space for conductors. For high voltage machines use small value of ampere conductors per meter.

*Size of machine:* in large size machine there is more space for accommodating copper. Therefore high value of ‘ac’ could be used.
Armature reaction: if using high value of ‘ac’, armature mmf becomes high. This means under loaded condition there will be greater distortion of field form resulting in a large reduction in the value of flux. To compensate this field ampere turns are needed to be increased. Thus overall cost of copper in the machine will increase.

Commutation: a high value of ‘ac’ means either ampere conductors used are more or diameter is small. Reactance voltage increases with high ampere conductors. With small diameter, deeper slots are used. Deeper slots also give higher reactance voltage. Higher reactance voltage results in bad commutation. Thus using higher ‘ac’ affects the commutation badly.

The value of ‘ac’ varies from 15000 to 50000 ampere conductors per meter.
Core length:
Factors affecting the length of core:

i) Cost: the manufacturing cost of a machine with large core length, is less. This is because the proportion of inactive copper to active copper is smaller for greater the length of core. Therefore it is desirable to have large core length for less cost.

ii) Ventilation: the ventilation of large core length is difficult because the central portion of the core tends to attain a high temperature rise. If long armature are necessary special means for ventilation of core must be provided.

Limiting value of core length: the emf induced in a conductor should exceed $7.5/T_c N_c$ in order that the maximum value at load between adjacent segments limited to 30 V.
The voltage in a conductor at no load $e_z = B_{av} L V_a$

For a limiting case: $B_{av} L V_a = 7.5/T_c N_c$

Limiting value of core length $L = 7.5/(B_{av} V_a T_c N_c)$

$B_{av} = \text{average gap density } \text{wb/m}^2$

$V_a = \text{peripheral speed, m/s}$

$T_c = \text{turns per coil}$

$N_c = \text{number of coils between adjacent segments}$

**Armature diameter:**

The peripheral speed lies between 15 to 50 m/s. As the diameter of the armature increases, the peripheral velocity of the armature $v = \pi DN/60$ m/s, centrifugal force and its effects increases. Therefore the machine must be mechanically made robust to withstand the effect of centrifugal force. This increases the cost of the machine. In general for normal construction, peripheral velocity should not be greater than 30 m/s as for as possible.
\textit{Limiting value of armature diameter:}

Output \quad P = E I_a \times 10^{-3} \text{ kW}

\begin{align*}
E &= \text{emf per conductor} \times \text{conductors per parallel path} \\
&= e_z \frac{Z}{a} \\
P &= (e_z \frac{Z}{a}) I_a \times 10^{-3} = e_z (I_z \cdot \frac{Z}{a}) \times 10^{-3} \\
&= e_z \pi D \text{ac} \times 10^{-3}
\end{align*}

\[ D = \frac{(P \times 10^{-3})}{(\pi \text{ac} e_z)} \]
Selection of number of poles

Factors affecting the number of poles:

1. **Frequency**: As the number of poles increases, frequency of the induced emf $f = \frac{120}{PN}$ increases, core loss in the armature increases and therefore efficiency of the machine decreases.

2. **Weight of the iron used for the yoke**: Since the flux carried by the yoke is approximately $\Phi/2$ and the total flux $\Phi_T = p\Phi$ is a constant for a given machine, flux density in the yoke

$$B_y = \frac{\phi/2}{\text{cross sectional area of the yoke } A_y} = \frac{\phi_T}{2PA_y} \propto \frac{1}{P A_y}$$

It is clear that $A_y \propto 1/P$

as $B_y$ is also almost constant for a given iron. Thus, as the number of poles increases, $A_y$ and hence the weight of iron used for the yoke reduces.
3. **Weight of iron used for the armature core (from the core loss point of view):**

Since the flux carried by the armature core is $\phi/2$, eddy current loss in the armature core is

$$\propto B_c^2 f^2$$

$$\propto \left[ \frac{\phi/2}{A_c} \right]^2 f^2 \propto \left[ \frac{\Phi_T}{2 PA_c} \right]^2 \times \left[ \frac{PN}{120} \right]^2$$

$$\propto \frac{1}{A_c^2}$$ is independent of the number of poles.

On the other hand, since the hysteresis loss in the armature core is

$$\propto B_c^{1.6} f \propto \left( \frac{\Phi_T}{2 PA_c} \right)^{1.6} \times \frac{PN}{120} \propto \frac{1}{P^{0.6} A_c^{1.6}}$$

$$\propto \frac{1}{P^{0.6/1.6}}$$ decreases as the number of poles increases for a given hysteresis loss. Thus the weight of iron used for the armature core reduces as the number of poles increases.
4. Weight of overhang copper

For a given active length of the coil, overhang $\propto$ pole pitch $\pi \frac{D}{P}$ goes on reducing as the number of poles increases. As the overhang length reduces, the weight of the inactive copper used at the overhang also reduces.

AF: Active length of the coil
ABC or ADE: Overhang or inactive part of the coil
$\tau_4$: Pole pitch in case of 4 pole machine
$\tau_6$: Pole pitch in case of 6 pole machine
5. Armature reaction

Since the flux produced by the armature $\phi_a = \frac{AT_a}{\text{pole}}$ and armature ampere turns $AT_a$ / pole $= \frac{I_a Z}{2 A P}$ is proportional to $1 / P$, $\phi_a$ reduces as the number of poles increases. This in turn reduces the effect of armature reaction.

6. Overall diameter

When the number of poles is less, $AT_a$ / pole and hence the flux, produced by the armature is more. This reduces the useful flux in the air gap. In order to maintain a constant value of air gap flux, flux produced by the field or the field ampere-turns must be increased. This calls for more field coil turns and size of the coil defined by the depth of the coil $d_f$ and height of the coil $h_f$ increases. In order that the temperature rise of the coil is not more, depth of the field coil is generally restricted. Therefore height of the field coil increases as the size of the field coil or the number of turns of the coil increases. As the pole height, is proportional to the field coil height, height of the pole and hence the overall diameter of the machine increases with the increase in height of the field coil.

Obviously as the number of poles increases, height of the pole and hence the overall diameter of the machine decreases.
7. **Length of the commutator**

Since each brush arm collects the current from every two parallel paths, current / brush arm \( = 2 I_a / A \) and the cross sectional area of the brush / arm 

\[
A_b = 2I_a / A \delta_b = 2I_a / P \delta_b
\]

\( \propto 1 / P \)

reduces as the number of poles increases.

As \( A_b = t_b w_b n_b \) and \( t_b \) is generally held constant from the commutation point of view, \( w_b n_b \) reduces as \( A_b \) reduces. Hence the length of the commutator 

\[
L_c = (w_b n_b + \text{clearances})
\]

reduces as \( A_b \) reduces or the number of poles increases.

\( w_b \) – width of the brush, \( t_b \) – thickness of the brush, \( n_b \) – number of brushes per spindle.
8. **Flash over**
As the number of poles increases, voltage between the segments
\[ E_b = \frac{\text{voltage between positive and negative brushes}}{\text{number of segments / pole}} \]
increases. Because of the increased value of \( E_b \) and carbon dust collected in the space where the mica is undercut, chances of arcing between commutator segments increases. The arc between the segments in turn may bridge the positive and negative brushes leading to a dead short circuit of the armature or flash over.

9. **Labour charges**
As the number of poles increases cost of labour increases as more number of poles are to be assembled, more field coils are to be wound, placed on to the pole, insulate, interconnect etc.
Length of air gap:

i) **Armature reaction**: to prevent excessive distortion of field form by armature reaction the field mmf must be large as compare to armature mmf. A machine designed with long air gap requires large field mmf. Thus the distortion effect of armature reaction can be reduced by large air gap length.

ii) **Circulating current**: if air gap length is small, a slight irregularity in the air gap would result large circulating current.

iii) **Noise**: the operation of machine with large air gap length is comparatively quite.

iv) **Cooling**: machine with large air gap length have better ventilation.

v) **Pole face losses**: if the length of air gap is made large, the variation in air gap flux density due to slotting are small. Therefore pulsation loss in the pole faces decreases.
**Estimation of air gap length:**

Mmf required for air gap $AT_g = 800000 B_g K_g l_g$

And armature mmf per pole $AT_a = ac\tau/2$

The value of gap mmf is normally between 0.5 to 0.7 of armature mmf. The usual value is 0.55.

$AT_g = (0.5 \text{ to } 0.7) \quad AT_a = (0.5 \text{ to } 0.7) \quad ac\tau/2$

From above equations $lg = (0.5 \text{ to } 0.7) \quad ac\tau/1600000K_gB_g$

Gap contraction factor $K_g$ may assumed as 1.15.

Usually the value of air gap length lies between 0.01 to 0.015 of pole pitch.
ARMATURE WINDING

The armature winding can broadly be classified as concentrated and distributed winding. In case of a concentrated winding, all the conductors / pole is housed in one slot. Since the conductors / slot is more, quantity of insulation in the slot is more, heat dissipation is less, temperature rise is more and the efficiency of operation will be less. Also emf induced in the armature conductors will not be sinusoidal. Therefore:

a. design calculations become complicated (because of the complicated expression of non-sinusoidal wave).

b. Core loss increases (because of the fundamental and harmonic components of the non-sinusoidal wave) and efficiency reduces.

c. Communication interference may occur (because of the higher frequency components of the non-sinusoidal wave).

Hence no concentrated winding is used in practice for a DC machine armature.

In a distributed winding (used to overcome the disadvantages of the concentrated winding), conductors / pole is distributed in more number of slots. The distributed winding can be classified as single layer winding and double layer winding.
In a single layer winding, there will be only one coil side in the slot having any number of conductors, odd or even integer depending on the number of turns of the coil. In a double layer winding, there will be 2 or multiple of 2 coil sides in the slot arranged in two layers. Obviously conductors / slot in a double layer winding must be an even integer.

Since for a given number of conductors, poles and slots, a single layer winding calls for less number of coils of more number of turns, reactance voltage proportional to \((\text{turn})^2\) is high. This decreases the quality of commutation or leads to sparking commutation. Hence a single layer winding is not generally used in DC machines. However it is much used in alternators and induction motors where there is no commutation involved.

Since a double layer winding calls for more number of coils of less number of turns/coil, reactance voltage proportional to \((\text{turn})^2\) is less and the quality of commutation is good. Hence double layer windings are much used in DC machines. Unless otherwise specified all DC machines are assumed to be having a double layer winding. A double layer winding can further be classified as simplex or multiplex and lap or wave winding.
Number of armature conductors

The generated emf in the armature

\[ E = V + I_a R_m \] for generator

\[ E = V - I_a R_m \] for motor

where \( V \) = terminal voltage and

\( R_m \) = sum of voltage drop in the armature winding, inter-pole winding, series winding and brush contact drop

i) For large 500 volt machine \( I_a R_m = 2 \) to \( 2.5\% \) of terminal voltage

ii) For small 250 volt machine \( I_a R_m = 5 \) to \( 10\% \) of terminal voltage

Total number of conductors in series \( Z_c = E/\text{mean emf per conductor} \)

\[ = E/e_z \]

For a simplex lap winding \( Z_c \) represent total number of armature conductor per pole. \((A=P)\)

For a simplax wave winding \( Z_c \) represent half the total number of conductor on the armature irrespective of number of poles. \((A=2)\)
CROSS-SECTIONAL AREA OF THE ARMATURE CONDUCTORS

Since the armature conductors are connected in series in any of the parallel paths of dc machine, maximum value of the current to be carried by the conductor is \( I_a / A \) and the cross sectional area of the conductor \( a = I_a / A \delta \, \text{mm}^2 \). The current density \( \delta \) lies between 4.5 and 7.0 \( \text{A} / \text{mm}^2 \) in practice.

As the current density increases, cross sectional area decreases, resistance of the conductor increases, \( I^2R \) loss increases and temperature of the machine increases. If the cooling facility is not good, temperature rise will unnecessarily be high. Hence higher value of current density should be used in machines where the peripheral velocity or speed of operation is high or where cooling facility is good.
Number of armature slots:
The following factors are to be considered while selecting the number of slots:

1. *Flux pulsations:* flux pulsation means changes in the air gap flux because of changes in the air gap reluctance between the pole faces and irregular armature core surface. Flux pulsation losses rise to eddy current losses and produce magnetic noise. The flux pulsations are reduced with increased number of slots.

2. *Cooling:* for large number of slots, lesser number of conductors per slot therefore, cooling is better.

3. *Commutation:* for commutation point of view, large number of slots and smaller number of conductors per slot are better.
4. *Tooth width*: for large number of slots the slot pitch reduces and also the tooth width. With reduction in tooth width flux density at the minimum section of tooth increases causing increase in iron losses.

5. *Cost*: cost of punching slots in stampings increases with the number of slots to be punched.